

Exercise Set 6: More on the Finite Difference Methods

Exercise 1 (Non-conservative scheme)

Consider the non-conservative scheme for Burgers' equation:

$$U_j^{n+1} = U_j^n - \frac{k}{h} U_j^n (U_j^n - U_{j-1}^n). \quad (1)$$

Show that this scheme maps non-negative monotonically increasing sequences onto non-negative monotonically increasing sequences. *Remark: while this scheme is not conservative, is still of interest in a few contexts. The non-negativity condition is due to the upwinding implicit in the scheme.*

Exercise 2 (Monotonicity of FTCS)

What is the flux of the FTCS scheme for the transport equation? Relate it to other fluxes that you know. Determine whether the FTCS scheme is monotone.

Exercise 3 (Order of Convergence)

Show that the FTFS scheme for the transport equation is of order $(1,1)$ and that the FTCS scheme is of order $(1,2)$.

Exercise 4 (Linear Scheme)

Under what conditions is a linear scheme $F(U, V) = w_1 U + w_2 V$ a monotone scheme? Describe the discrete Kruzkov entropy-entropy-flux pairs.

Exercise 5 (Roe flux)

Show that the Roe flux can be written as

$$F(U, V) = \frac{f(V) + f(U)}{2} + \frac{1}{2} \operatorname{sgn}(V - U) |f(V) - f(U)|. \quad (2)$$

Either show that the Roe flux is monotonicity-preserving or give a counter example.